```
> ### Doroshin A.V.: Examples of solving linear algebra tasks
>
> restart;
> with(linalg):
Warning, new definition for norm
Warning, new definition for trace
> ###--------------------------------------------------------------
  -------------------
> ### GENERAL ACTIONS:
> ### Input of matrix, arrays, vectors:
> A:=array(1..6,1..6,[[0.500,0.22,0.33,2,0.4,0.6],[44,0.155,66,6.3
  ,4.4,5.6],[77,88,0.199,1.1,5.3,7.3],[4,55,2.3,5.1,4.2,2.2],[8.1,
  2.2,3,3,5.3,6.3],[4.5,2.2,1,3,7,5.5]]);C:=matrix([[1,2,3],[4,5,6
  ],[7,8,9]]);Y:=matrix([[1,2,3,4,5,6],[7,8,9,10,11,12],[13,14,15,
  16,17,18],[19,20,21,22,23,24],[25,26,27,28,29,30],[31,32,33,34,3
  5,36]]);
```

$$A := \begin{bmatrix} .500 & .22 & .33 & 2 & .4 & .6 \\ 44 & .155 & 66 & 6.3 & 4.4 & 5.6 \\ 77 & 88 & .199 & 1.1 & 5.3 & 7.3 \\ 4 & 55 & 2.3 & 5.1 & 4.2 & 2.2 \\ 8.1 & 2.2 & 3 & 3 & 5.3 & 6.3 \\ 4.5 & 2.2 & 1 & 3 & 7 & 5.5 \end{bmatrix}$$

$$C := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$Y := \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 7 & 8 & 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 & 17 & 18 \\ 19 & 20 & 21 & 22 & 23 & 24 \\ 25 & 26 & 27 & 28 & 29 & 30 \\ 31 & 32 & 33 & 34 & 35 & 36 \end{bmatrix}$$

```
> b:=vector([5,4,3,0.2,5.3,3.1]);b_:=b;
```

$$b := [5, 4, 3, .2, 5.3, 3.1]$$

$$b\_ := b$$

```
> B:=matrix([[aa,bb,cc],[dd,ee,ff],[gg,hh,ii]]);d:=vector([PP,QQ,R
  R]);
```

$$B := \begin{bmatrix} aa & bb & cc \\ dd & ee & ff \\ gg & hh & ii \end{bmatrix}$$

$$d := [PP, QQ, RR]$$

```
>
> ### Solving linear systems A*x=b and B*x=d:
> x:=linsolve(A,b);
```

$$x := [.2461187558, -.1807036876, -.3139609226, 2.609984862, -.9887092131, .3263608250]$$

```
> linsolve(B,d);
```

$$\left[-\frac{-bb\ ii\ QQ + bb\ RR\ ff - cc\ ee\ RR + cc\ hh\ QQ - PP\ hh\ ff + PP\ ee\ ii}{\%1},\ \frac{gg\ cc\ QQ - gg\ PP\ ff - cc\ dd\ RR - aa\ ii\ QQ + aa\ RR\ ff + PP\ dd\ ii}{\%1},\ \frac{ee\ gg\ PP - aa\ ee\ RR + aa\ hh\ QQ - hh\ dd\ PP - gg\ bb\ QQ + dd\ bb\ RR}{\%1} \right]$$

$$\%1 := aa\ hh\ ff - aa\ ee\ ii - gg\ bb\ ff + ee\ gg\ cc - hh\ dd\ cc + dd\ bb\ ii$$

```
> 
> ### Determinants and ranks evaluations:
> det(A);det(B);det(C);det(Y);rank(A);rank(B);rank(C);rank(Y);
```

$$.5653391689\ 10^{7}$$

$$aa\ ee\ ii - aa\ hh\ ff - dd\ bb\ ii + hh\ dd\ cc + gg\ bb\ ff - ee\ gg\ cc$$

$$0$$

$$0$$

$$6$$

$$3$$

$$2$$

$$2$$

```
> 
> ### Transposing the matrix:
> Atr:=transpose(A);
```

$$Atr := \begin{bmatrix} .500 & 44 & 77 & 4 & 8.1 & 4.5 \\ .22 & .155 & 88 & 55 & 2.2 & 2.2 \\ .33 & 66 & .199 & 2.3 & 3 & 1 \\ 2 & 6.3 & 1.1 & 5.1 & 3 & 3 \\ .4 & 4.4 & 5.3 & 4.2 & 5.3 & 7 \\ .6 & 5.6 & 7.3 & 2.2 & 6.3 & 5.5 \end{bmatrix}$$

```
> 
> ### Multiplications of Matrices: DDD=A*Y and AAA=Atr*A:
> DDD:=evalm(A &* Y); AAA:=evalm(Atr &* A);
```

$$DDD := \begin{bmatrix} 72.930 & 76.980 & 81.030 & 85.080 & 89.130 & 93.180 \\ 1306.385 & 1432.840 & 1559.295 & 1685.750 & 1812.205 & 1938.660 \\ 1075.287 & 1254.186 & 1433.085 & 1611.984 & 1790.883 & 1969.782 \\ 689.0 & 761.8 & 834.6 & 907.4 & 980.2 & 1053.0 \\ 447.3 & 475.2 & 503.1 & 531.0 & 558.9 & 586.8 \\ 435.4 & 458.6 & 481.8 & 505.0 & 528.2 & 551.4 \end{bmatrix}$$

$$AAA := \begin{bmatrix} 7967.110000 & 7030.65000 & 2957.48800 & 421.100 & 693.1300 & 893.3800 \\ 7030.65000 & 10778.75243 & 163.1146 & 391.9165 & 725.2300 & 790.3600 \\ 2957.48800 & 163.1146 & 4371.438501 & 440.4089 & 324.1467 & 400.7107 \\ 421.100 & 391.9165 & 440.4089 & 88.91 & 92.67 & 91.13 \\ 693.1300 & 725.2300 & 324.1467 & 92.67 & 142.34 & 144.70 \\ 893.3800 & 790.3600 & 400.7107 & 91.13 & 144.70 & 159.79 \end{bmatrix}$$

```
> 
> ### Inverting the matrix: Aobr:=A^-1:
```

```
> Aobr:=evalm(A^(-1));
```

$$Aobr :=$$

[.06822708227 , .002100225765 , .01561504882 , -.02471896643 , -.04689256950 , .03329409891]

[-.04106396027 , -.0009634831262 , -.0008142740848 , .01981085152 , .01785933380 , -.02183992615]

[-.07511633700 , .01456107805 , -.009237813068 , .01515501159 , .01680803273 , -.01968506022]

[.5921991838 , .0007084399248 , .003337986954 , -.005519808344 , -.08767868257 , .03288458180]

[-.06755273417 , .01221017579 , .01017792985 , -.01789201984 , -.3952656375 , .4413439506]

[-.2627805878 , -.01990708753 , -.02554508607 , .03532727779 , .5790570011 , -.4127548995]

```
> ### Aobr*A is the identity matrix:
> isitE:=evalm(Aobr &* A);evalm(A &* Aobr);
```

$$isitE :=$$

$$\begin{bmatrix} 1.000000000 & -.30\,10^{-9} & .1\,10^{-10} & -.7\,10^{-10} & 0 & 0 \\ 0 & .9999999995 & -.1\,10^{-10} & .2\,10^{-10} & -.1\,10^{-9} & 0 \\ -.3\,10^{-10} & .3\,10^{-10} & .9999999998 & -.1\,10^{-10} & 0 & 0 \\ 0 & -.4\,10^{-10} & 0 & 1.000000001 & .1\,10^{-9} & 0 \\ .1\,10^{-8} & .1\,10^{-9} & -.2\,10^{-9} & 0 & 1.000000000 & 0 \\ 0 & -.9\,10^{-9} & -.5\,10^{-9} & -.1\,10^{-8} & -.1\,10^{-8} & 1.000000000 \end{bmatrix}$$

$$\begin{bmatrix} 1.000000000 & 0 & .1\,10^{-10} & -.2\,10^{-10} & .1\,10^{-9} & -.1\,10^{-9} \\ 0 & .9999999998 & .3\,10^{-9} & -.3\,10^{-9} & 0 & -.1\,10^{-8} \\ 0 & -.1\,10^{-9} & 1.000000001 & -.2\,10^{-9} & -.1\,10^{-8} & 0 \\ -.7\,10^{-9} & .1\,10^{-10} & .5\,10^{-10} & 1.000000000 & -.1\,10^{-8} & .3\,10^{-9} \\ 0 & 0 & .1\,10^{-9} & -.1\,10^{-9} & 1.000000000 & -.1\,10^{-8} \\ 0 & 0 & 0 & -.1\,10^{-9} & 0 & 1.000000000 \end{bmatrix}$$

```
>
> ### Eigenvalues evaluating:
> eigenvalues(AAA);
```

1.057225862, 2.802106335, 142.8935891, 804.2279118, 5505.600224, 17051.75987

```
>
> ### Eigenvectors evaluating: [eigenvalue, multiplicity,
  {[eigenvector]}]:
> eigenvectors(AAA);
```

[2.802105323, 1, {

[.06981031838, -.04480553902, -.09912529689, .8625302158, -.4882564429, .03066643724]

}], [142.8935871, 1, {

[.01899299205, .09102133496, .1306540084, -.3475117301, -.6849957037, -.6199196691]}]

```
, [1.057223817, 1, {
[-.08256936157, .04313396800, .05392436649, -.3137752527, -.5320841124, .7790028962]}
], [5505.600224, 1,
{[.3212940965, -.4692915804, .8184894516, .05887998147, .02983088954, .04747421990]}
], [17051.75987, 1,
{[.6415460073, .7420993600, .1644028797, .03807351807, .06211280019, .07328957992]}]
, [804.2279121, 1, {
[.6878494832, -.4657263091, -.5227261047, -.1786221407, -.06726921853, .01698517431]}
]
> 
> 
> ###-------------------------------------------------------------
  --------------------
> ###   Gauss method for solving system of linear equations
> n:=6;
```

$$n := 6$$

```
> ### Start of TRIANGULARIZATION down
> for k from 1 by 1 to n-1 do
> for i from k+1 by 1 to n do
> b[i]:=b[i]-A[i,k]*b[k]/A[k,k];
> for j from n by (-1) to k do
> A[i,j]:=A[i,j]-A[i,k]*A[k,j]/A[k,k];
> od; ### j-cycle
> od; ### i-cycle
> od; ### k-cycle
> ###-----TRIANGULARIZATION down is finished
> evalm(A);evalm(b);
```

$$\begin{bmatrix} .500 & .22 & .33 & 2 & .4 & .6 \\ 0 & -19.20500000 & 36.96000000 & -169.7000000 & -30.80000000 & -47.20000000 \\ 0 & 0 & 53.53287660 & -785.1173392 & -143.0948972 & -218.1103619 \\ 0 & 0 & 0 & 1016.363088 & 188.5869068 & 282.6238179 \\ 0 & 0 & 0 & 0 & 4.46629630 & 4.77564980 \\ 0 & 0 & 0 & 0 & 0 & -2.422745315 \end{bmatrix}$$

$$[5, -436.0000000, -1995.655038, 2558.472004, -2.8572835, -.790688938]$$

```
> 
> ### Start of TRIANGULARIZATION up
> for k from 1 by 1 to n-1 do
> R:=n-k+1:
> for i from n-k by -1 to 1 do
> b[i]:=b[i]-A[i,R]*b[R]/A[R,R];
> for j from n-k+1 by -1 to 1 do
> A[i,j]:=A[i,j]-A[i,R]*A[R,j]/A[R,R];
> od; ### j-cycle
> od; ### i-cycle
> od; ### k-cycle
```

```
> ###-----TRIANGULARIZATION up is finished
> evalm(A);evalm(b);
```

$$\begin{bmatrix} .500 & 0 & 0 & 0 & 0 & 0 \\ 0 & -19.20500000 & 0 & 0 & -.1\,10^{-7} & .1\,10^{-7} \\ 0 & 0 & 53.53287660 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1016.363088 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4.46629630 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2.422745315 \end{bmatrix}$$

$$[.1230594026, 3.47041515, -16.807233, 2652.692291, -4.415868072, -.790688938]$$

```
> ### SOLUTION by Gauss
> XGauss:=array(1..6);### empty array
```

$$XGauss := \text{array}(1 .. 6, [\ ])$$

```
> for i from 1 to n do XGauss[i]:=b[i]/A[i,i]; od;
```

$$XGauss_1 := .2461188052$$

$$XGauss_2 := -.1807037308$$

$$XGauss_3 := -.3139609539$$

$$XGauss_4 := 2.609984879$$

$$XGauss_5 := -.9887091620$$

$$XGauss_6 := .3263607335$$

```
> ### Comparison XGauss with x:
> evalm(x);evalm(XGauss);
```

$$[.2461187562, -.1807036880, -.3139609231, 2.609984864, -.9887092136, .3263608249]$$

$$[.2461188052, -.1807037308, -.3139609539, 2.609984879, -.9887091620, .3263607335]$$

```
>
>
>
>
> ### Cholesky expansion
> Tensor_inercii :=
  matrix([[A,-Ixy,-Ixz],[-Ixy,B,-Iyz],[-Ixz,-Iyz,C]]);
```

$$Tensor\_inercii := \begin{bmatrix} A & -Ixy & -Ixz \\ -Ixy & B & -Iyz \\ -Ixz & -Iyz & C \end{bmatrix}$$

```
> L := cholesky(Tensor_inercii);
```

$$L := \begin{bmatrix} \sqrt{A} & 0 & 0 \\ -\frac{Ixy}{\sqrt{A}} & \sqrt{B - \frac{Ixy^2}{A}} & 0 \\ -\frac{Ixz}{\sqrt{A}} & \frac{-Iyz - \frac{Ixz\ Ixy}{A}}{\sqrt{B - \frac{Ixy^2}{A}}} & \sqrt{C - \frac{Ixz^2}{A} - \frac{\left(-Iyz - \frac{Ixz\ Ixy}{A}\right)^2}{B - \frac{Ixy^2}{A}}} \end{bmatrix}$$

```
> Lt := transpose(L);
```

$$Lt := \begin{bmatrix} \sqrt{A} & -\frac{Ixy}{\sqrt{A}} & -\frac{Ixz}{\sqrt{A}} \\ 0 & \sqrt{B - \frac{Ixy^2}{A}} & \frac{-Iyz - \frac{Ixz\ Ixy}{A}}{\sqrt{B - \frac{Ixy^2}{A}}} \\ 0 & 0 & \sqrt{C - \frac{Ixz^2}{A} - \frac{\left(-Iyz - \frac{Ixz\ Ixy}{A}\right)^2}{B - \frac{Ixy^2}{A}}} \end{bmatrix}$$

```
> evalm(L &* Lt);
```

$$\begin{bmatrix} A & -Ixy & -Ixz \\ -Ixy & B & -Iyz \\ -Ixz & -Iyz & C \end{bmatrix}$$

```
>
>
>
```